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STATIC MULTIPLE-LOAD MEASUREMENT TECHNIQUE AS UTILIZED IN THE NAVAL SURFACE WEAPONS CENTER'S WIND TUNNELS

NAVAL SURFACE WEAPONS CENTER

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WHITE OAK LABORATORY

STATIC MULTIPLE-LOAD MEASUREMENT TECHNIQUE AS UTILIZED IN THE NAVAL SURFACE WEAPONS CENTER'S WIND TUNNELS

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This report describe~ the basic scheme used in the Naval Surface Weapons Center's wind tunnels for measuring static forces and moments on wind-tunnel models. The general fundamentals used in the design of balances are explained as well as how the calibrations are conducted and how the data are reduced. The basic technique consists of using multiple combined loads in the calibration of the balance and in sing an iterative approach to reduce the balance electrical output to aerodynamic loads.

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STATIC MULTIPLE-LOAD MEASUREMENT TECHNIQUE AS UTILIZED IN THE NAVAL SURFACE WEAPONS CENTER'S WIND TUNNELS

The purpose of this report is to document the techniques used at the Naval Surface Weapons Center in the measurement of static forces and moments on vehicle models in the wind tunnels.

This work was funded by the Strategic Systems Project Office under Task Number SSPO 77402.

R. W. SCHLIE
By direction

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INTRODUCTION

One of the primary uses for wind tunnels is to be able to place scale models of vehicles into the tunnel in order to measure the aerodynamic loads on the vehicle. This concept of testing small-scale vehicles has been done ever since the development of the first airplane. Early systems utilized strain-gage beam balances mounted outside of both the model and the tunnel. Later improvements in instrumentation made it possible to place a cantilever balance instrumented with strain gages inside the model.

The primary vehicles tested at the Naval Surface Weapons Center, White Oak Laboratory (formerly the Naval Ordnance Laboratory), have been weapon configurations that had relatively small out-of-plane loads in comparison to those in the plane of the angle of attack. Consequently, it was possible to utilize a system whereby the interactions between the in-plane and out-of-plane loads were nulled by electrically shunting appropriate arms of the Wheatstone bridge circuits (Ref. (1)). This is accomplished by loading the balance in one plane and then adjusting the resistances in the appropriate legs of the bridges in the gages of the orthogonal plane so that there is no indication of a load in this orthogonal plane. This technique effectively eliminated all first-order interactions and proved to be quite accurate and acceptable.

In recent years, with the introduction of re-entry vehicles and quite small tolerances on the measurement of in-plane and out-of-plane loads it became necessary to improve the accuracy of the load measurements. The scheme chosen to do this was one that has been utilized for some time by other tunnel complexes where a wider variety of vehicle types have been tested (Refs. (2)-(5)). The

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(1) Shantz, L, Gilbert, B. D., White, C.E.,"NOL Wind-Tunnel Internal Strain-Gage Balance System," NSWC/WOL, NAVORD Rpt. 2972, Sep 1953

(2) Hansen, Raymond M., "Evaluation and Calibration of Wire-Strain-Gage Wind Tunnel Balances Under Load," AGARD Rpt. 13, Feb 1956

(3) "Calibration and Evaluation of Multicomponent Strain-Gage Balances," Prepared for presentation to the NASA Interlaboratory Force Measurements Group meeting held at JPL 16-17 Apr 1974

(4) Smith, David L., "An Efficient Algorithm Using Matrix Methods to Solve Wind-Tunnel Force-Balance Equations," Langley Research Center, NASA TN D-6860, Aug 1972

(5) Hurlburt, R. T. and Berg, D. E., "A Non-Iterative Method of Reducing Wind Tunnel Moment Balance Data," Sandia Laboratories, Albuquerque, N. M., SC-RR-720665, Mar 1973

basic scheme reported on in Reference (4) has been modified and adapted to our needs. Basically this involves the use of second-order equations to fit for the interactions rather than the electrical nulling of only the first-order interactions.

STRAIN-GAGE BALANCE

Forces and moments can be measured by equating a change in strain at a point on a cantilever beam to the force and/or moment that caused the strain. The ideal situation is to construct a balance so that the tension or compression in an individual section is sensitive to only one type of loading. It is then possible to locate a strain gage on the surface of the balance and calibrate the change in resistance of the gage with respect to the load that caused it.

Rather than use one gage it is better to utilize four gages arranged in a Wheatstone bridge circuit as shown in Figure 1.

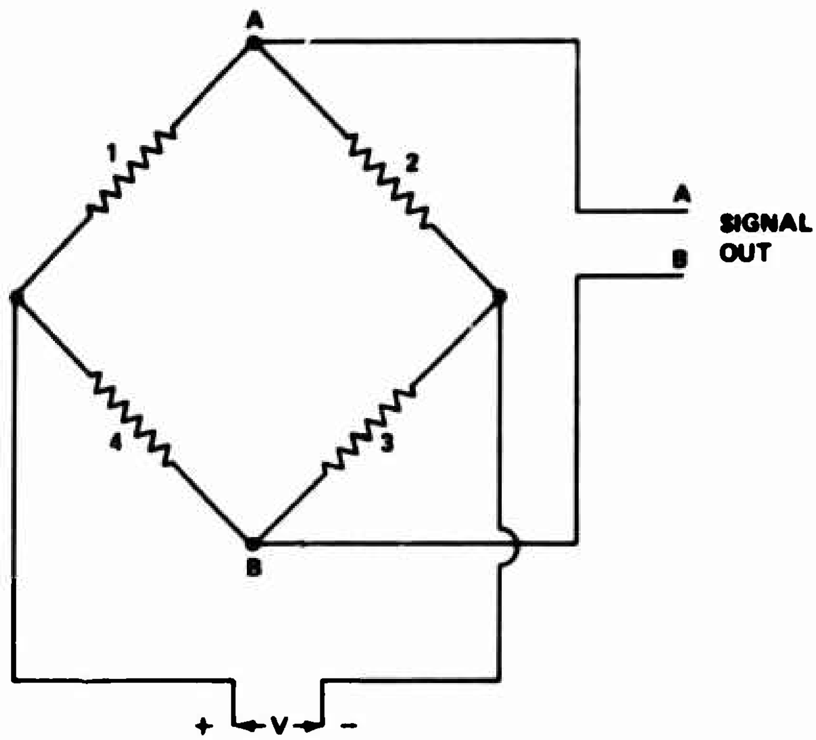

FIG. 1 WHEATSTONE BRIDGE

This design makes it possible to measure the absolute change in voltage, makes the output proportional to four times the output of one gage, helps to minimize temperature effects, and helps to isolate the load. If the four gages in the bridge are located on a balance section so that the load causes the surface under gages 1 and 3 to be in tension (increase in gage resistance) and the surface under gages 2 and 4 to be in compression (decrease in gage resistance) signal lead A would be at a lower potential and signal B at a higher potential. By means of a calibration this change in potential can be equated to the load that caused it.

In order to measure a force and the moment it causes about some point on the balance, it is necessary to measure the strain in the balance at two locations. For instance, in Figure 2, if a force is applied at any point along the beam, it generates a moment which varies along the beam.

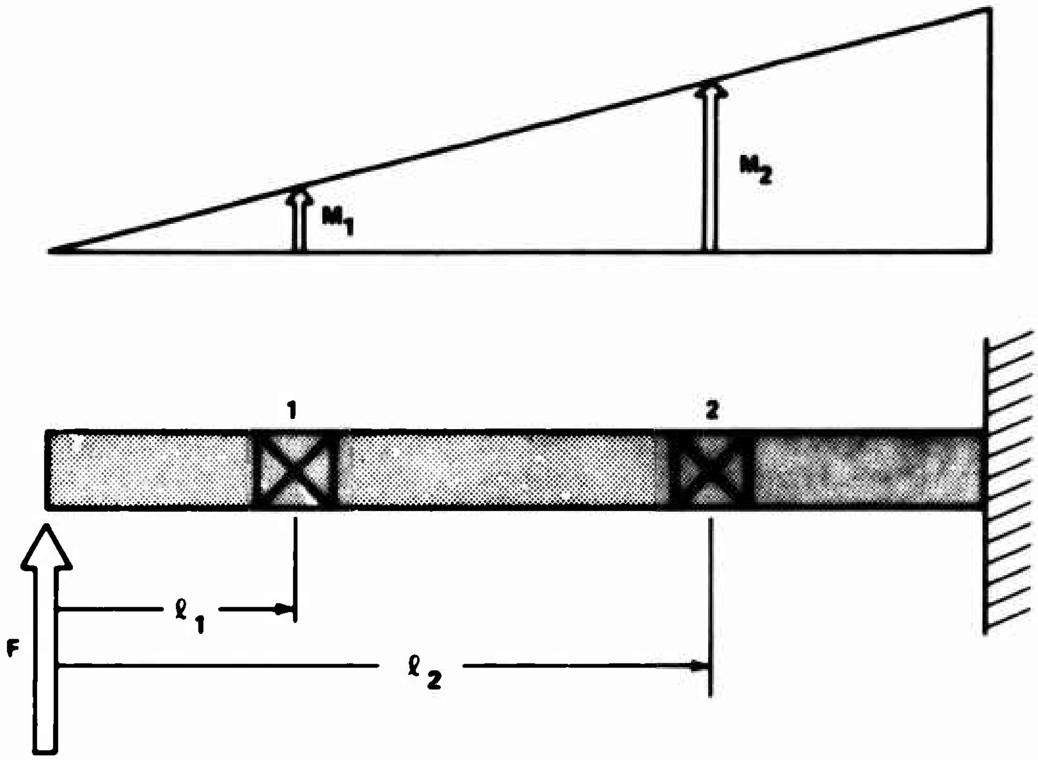

FIG. 2 CANTILEVER BEAM BALANCE

The moments at gage sections one and two are equal to,

$$M_1 = F\ell_1$$

$$M_2 = F\ell_2 \quad .$$

These moments can be measured with Wheatstone bridges located at sections one and two. Assuming that these are measured, the moments can be added to get

$$M_1 + M_2 = F(\ell_1 + \ell_2) \tag{1}$$

or they can be subtracted to get

$$M_1 - M_2 = F(\ell_1 - \ell_2) \quad . \tag{2}$$

Since,

$$\ell_1 - \ell_2 = \Delta X = \text{known distance between the gage sections,}$$

the equations can be written as

$$M_1 + M_2 = F(\Delta X + 2\ell_2) \tag{3}$$

$$M_1 - M_2 = F\Delta X \quad . \tag{4}$$

These can then be solved for the two unknowns, F and ℓ_2.

$$F = \frac{M_1 - M_2}{\Delta X} \tag{5}$$

$$\ell_2 = \frac{M_1 + M_2}{2F} - \frac{\Delta X}{2} \tag{6}$$

Knowing these, the force and its point of application, the moment about any point on the balance can be calculated.

Utilizing the above concepts, it is possible to instrument the balance in both the pitch and yaw planes. Since both planes are instrumented the same, only the pitch plane will be shown. There are two different wiring circuits in use at the White Oak Laboratory. In the first, the two-moment method, a forward balance section and an aft section are wired independently as shown in Figure 3. By treating the fore and aft sections separately it is possible to minimize the effects of a temperature gradient along the balance. In this case it is necessary to add and subtract the moments measured at each section for use in Equations (5) and (6). This is done in the data-reduction program.

In the second wiring circuit, the force-moment method, the moments are added and subtracted electrically on the balance as is shown in Figure 4. This has the advantage in that the output signals are directly proportional to a force and a moment but has the disadvantage in that a temperature gradient along the balance causes a change in the output that is hard to adjust or account for in a rational manner.

Similar techniques are used to measure the roll moment and axial force, although the mechanical design of the gage mounting sections becomes more intricate. As shown in Figure 5, the roll section usually consists of a cruciform section, and the axial section of a series of thin webs.

As mentioned earlier, the objective of the balance designer is to design gage sections that will respond to only one type of load with no interactions between loads. If this could be accomplished, a large part of the rest of this report would not be necessary; but,

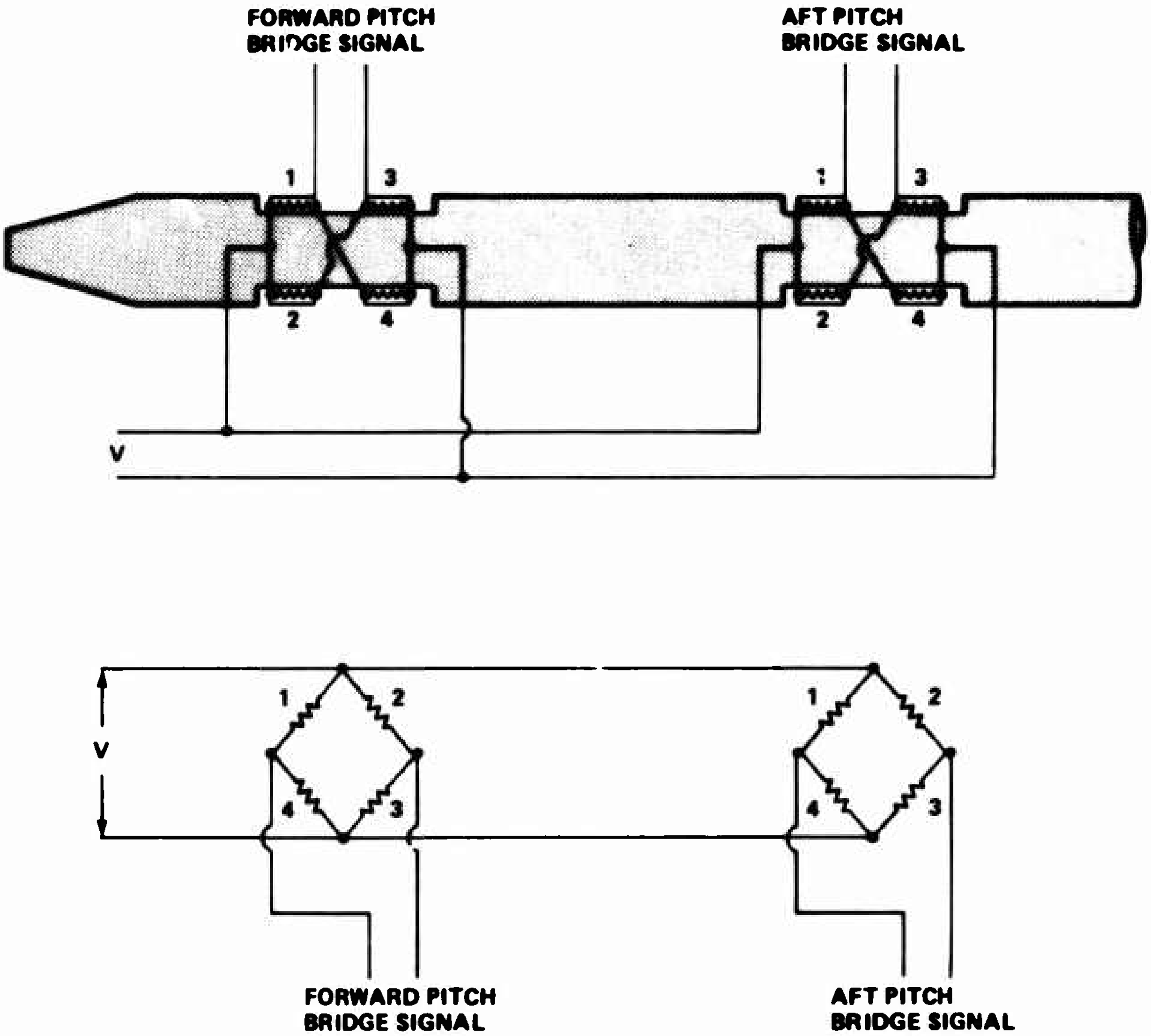

FIG. 3 TWO MOMENT PITCH GAGE SECTIONS

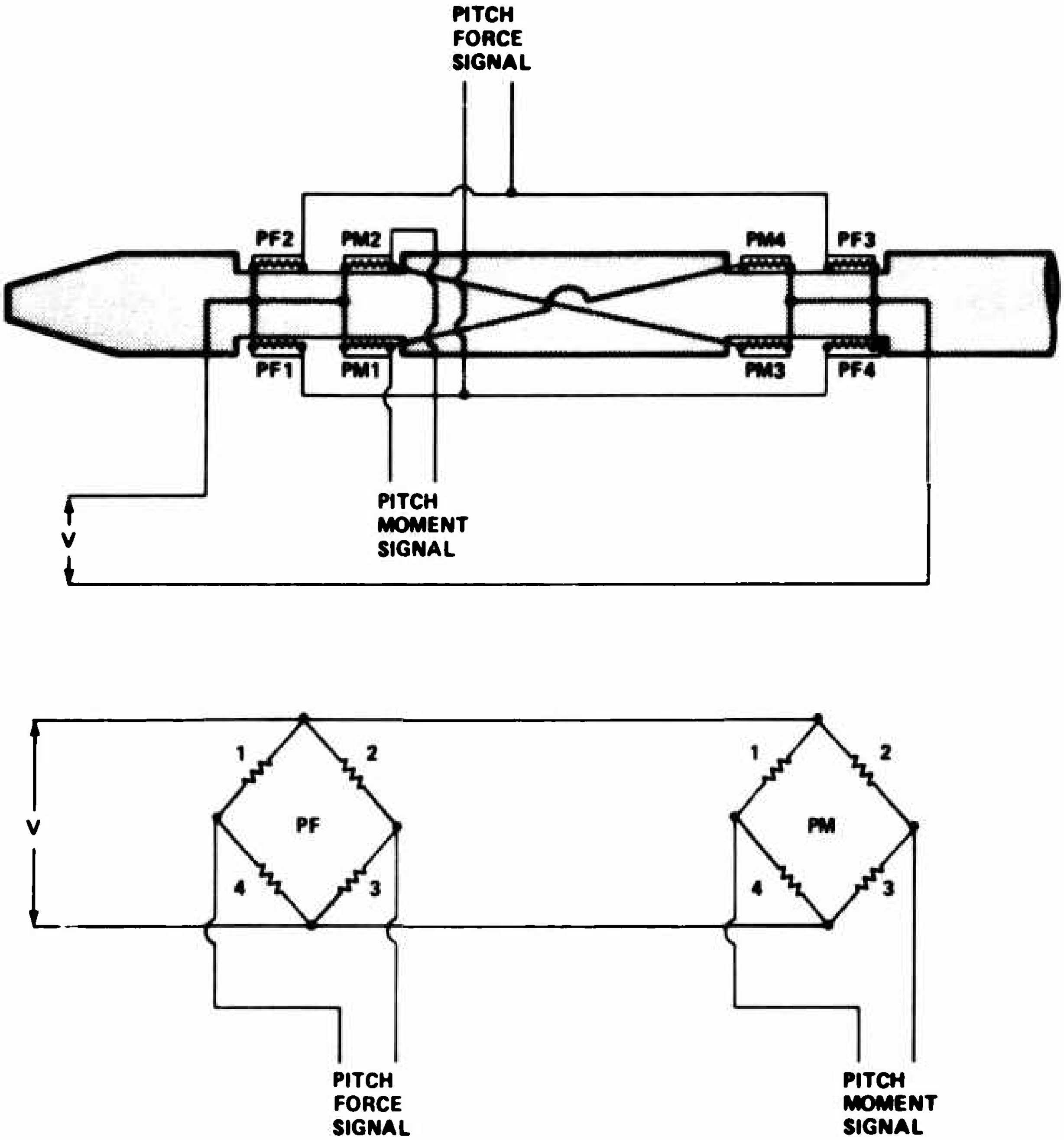

FIG. 4 FORCE-MOMENT PITCH GAGE SECTIONS

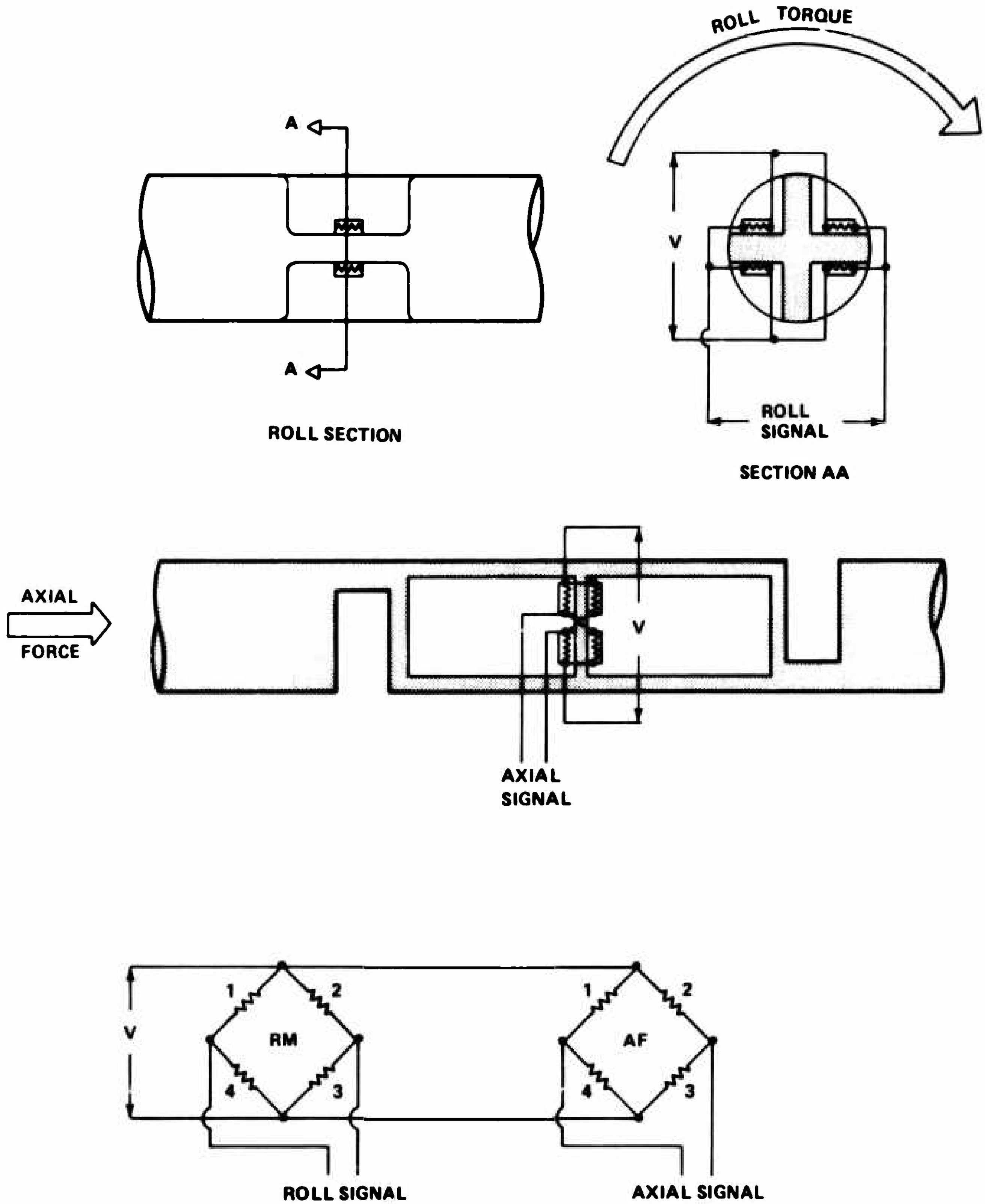

FIG. 5 ROLL AND AXIAL SECTIONS

since there are always some interactions (although they are usually very small) it is necessary to account for them in the resolution of the loads being measured.

GENERAL LOAD RESOLUTION TECHNIQUE

The general scheme in using a strain-gage balance to measure aerodynamic loads consists of first loading the balance with known loads and then relating its response to these known loads through a set of calibration constants. These constants can then be used to determine an unknown load applied to the balance by going through the inverse process. Since there are six degrees of freedom in the model's system, it is necessary to use a six-component balance to determine all of them. These six loads are:

F_N = normal force

F_Y = yaw force

M_Y = pitching moment

M_Z = yawing moment

M_X = rolling moment

F_A = axial force

The standard model axes and loads are shown in Figure 6.

If the assumption is made that the variation of the balance output with load is only slightly nonlinear, then only second-order load effects need to be calibrated (this assumption is checked during each calibration). All possible second-order load combinations are as follows:

$$
\begin{array}{lllllll}
F_N & F_N^2 & & & & & \\
F_Y & F_N F_Y & F_Y^2 & & & & \\
M_Y & F_N M_Y & F_Y M_Y & M_Y^2 & & & \\
M_Z & F_N M_Z & F_Y M_Z & M_Y M_Z & M_Z^2 & & \\
M_X & F_N M_X & F_Y M_X & M_Y M_X & M_Z M_X & M_X^2 & \\
F_A & F_N F_A & F_Y F_A & M_Y F_A & M_Z F_A & M_X F_A & F_A^2
\end{array}
$$

For bookkeeping purposes, the 27 load combinations have been assigned the following subscripts:

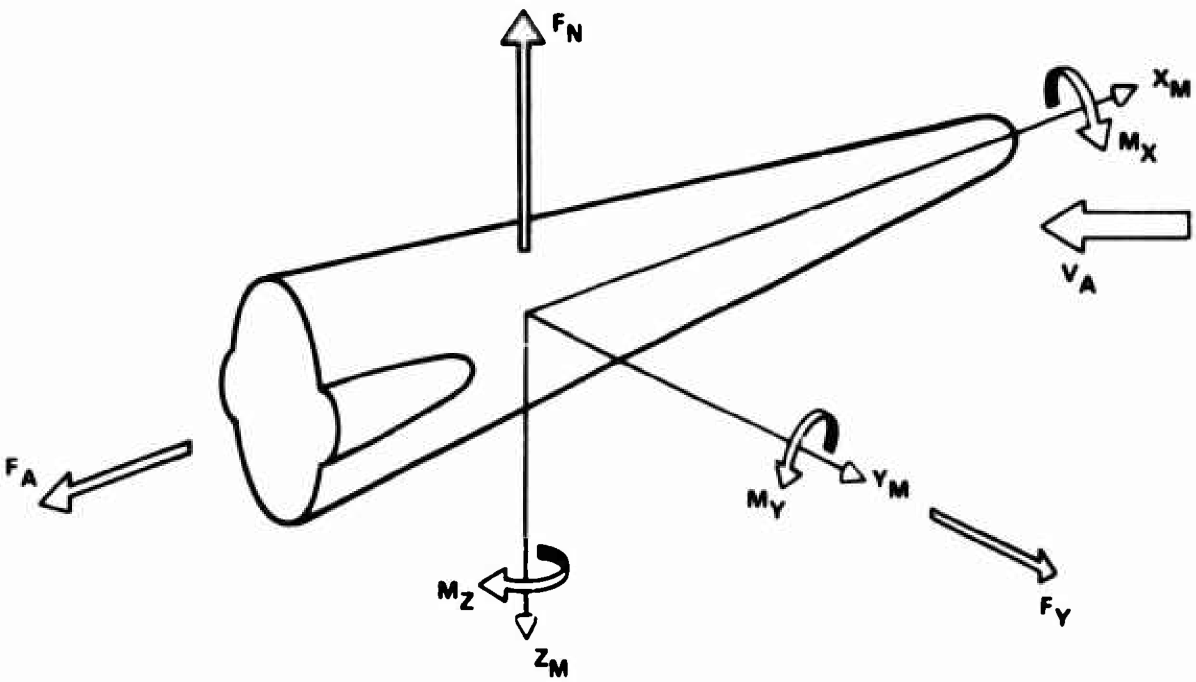

FIG. 6 MODEL AXES

Load	Subscript	Load	Subscript
F_N	1	$F_Y M_Z$	15
F_Y	2	$F_Y M_X$	16
M_Y	3	$F_Y F_A$	17
M_Z	4	M_Y^2	18
M_X	5	$M_Y M_Z$	19
F_A	6	$M_Y M_X$	20
F_N^2	7	$M_Y F_A$	21
$F_N F_Y$	8	M_Z^2	22
$F_N M_Y$	9	$M_Z M_X$	23
$F_N M_Z$	10	$M_Z F_A$	24
$F_N M_X$	11	M_X^2	25
$F_N F_A$	12	$M_X F_A$	26
F_Y^2	13	F_A^2	27
$F_Y M_Y$	14		

The six output channels of the balance have been designated as,

Channel	Load	Symbol
1	F_N	θ_1
2	F_Y	θ_2
3	M_Y	θ_3
4	M_Z	θ_4
5	M_X	θ_5
6	F_A	θ_6

According to the assumption that the highest order of non-linearity in the balance output is second order, the output of the i^{th} channel can be represented as

$$\theta_i = k_{i,1}F_N + k_{i,2}F_Y + \ldots + k_{i,14}F_YM_Y + \ldots + k_{i,27}F_A^2 \tag{7}$$

where $k_{i,j}$ $(i = 1,2,\ldots 6)$, $(j = 1,2,\ldots 27)$ are balance coefficients to be determined in the calibration of the balance. Equation (7) can thus be normalized by dividing both sides by $k_{i,i}$ and then expressing it as follows.

$$(L_i)_u = K_{i,1}F_N + K_{i,2}F_Y + \ldots + K_{i,27}F_A^2 \tag{8}$$

where

$$K_{i,j} = \frac{k_{i,j}}{k_{i,i}} = \text{normalized interaction coefficient when } i \neq j$$

$$K_{i,j} = 1 \text{ when } i = j$$

$$\mu_i = 1/k_{i,i} = i^{th} \text{ component sensitivity}$$

$$(L_i)_u = \mu_i\theta_i = \text{uncorrected load on the } i^{th} \text{ component}$$

For the case where $i = 1$, Equation (8) becomes

$$(F_N)_u = F_N + K_{1,2}F_Y + \ldots + K_{1,27}F_A^2 \quad . \tag{9}$$

Equation (9) can then be arranged as

$$F_N = (F_N)_u - (K_{1,2}F_Y + \ldots + K_{1,27}F_A^2) \quad . \tag{10}$$

Equation (8) consists of six equations similar to Equation (10) which present the true loads as a function of the uncorrected, indicated loads and all of the first- and second-order interactions. These equations are:

$$F_N = (F_N)_u - (K_{1,2}F_Y + \ldots + K_{1,27}F_A^2) \tag{11}$$

$$F_Y = (F_Y)_u - (K_{2,1}F_N + K_{2,3}M_Y + \ldots + K_{2,27}F_A^2) \tag{12}$$

$$M_Y = (M_Y)_u - (K_{3,1}F_N + \ldots + K_{3,27}F_A^2) \tag{13}$$

$$M_Z = (M_Z)_u - (K_{4,1}F_N + \ldots + K_{4,27}F_A^2) \tag{14}$$

$$M_X = (M_X)_u - (K_{5,1}F_N + \ldots + K_{5,27}F_A^2) \tag{15}$$

$$F_A = (F_A)_u - (K_{6,1}F_N + \dots + K_{6,27}F_A^2) \tag{16}$$

The remaining problem is one of how to utilize these equations in order to obtain the true loads acting on a model during a test. Since all of the equations are coupled through the first- and second-order interactions, an iterative approach has been utilized to solve them. It is not absolutely necessary to use an iterative scheme (see Ref. (5)) but in order to handle a wide range of balances for which the relative magnitude of the nonlinear terms varies, it is the most accurate and quickest way of solving them.

Matrix relationships were used in order to expedite the handling of Equations (11)-(16) by making the following definitions.

Output channel matrix

$$\Theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \end{bmatrix} = [\theta_i] \qquad i = 1,2,\dots 6$$

Load matrix

$$L = \begin{bmatrix} F_N \\ F_Y \\ M_Y \\ M_Z \\ M_X \\ F_A \end{bmatrix} = [L_i] \qquad i = 1,2,\dots 6$$

Second order force and moment product matrix

$$P = \begin{bmatrix} F_N^2 \\ F_N F_Y \\ F_N M_Y \\ \cdot \\ \cdot \\ \cdot \\ F_A^2 \end{bmatrix} = L_i L_j \qquad \begin{matrix} i = 1,2,\dots 6 \\ j = i,i+1,\dots 6 \end{matrix}$$

Sensitivity matrix

$$\mu = \begin{bmatrix} \mu_{1,1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu_{2,2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu_{3,3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu_{4,4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu_{5,5} & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu_{6,6} \end{bmatrix}$$

First-order interaction coeff. matrix

$$C_1 = \begin{bmatrix} 1 & K_{1,2} & K_{1,3} & K_{1,4} & K_{1,5} & K_{1,6} \\ K_{2,1} & 1 & K_{2,3} & K_{2,4} & K_{2,5} & K_{2,6} \\ K_{3,1} & K_{3,2} & 1 & K_{3,4} & K_{3,5} & K_{3,6} \\ K_{4,1} & K_{4,2} & K_{4,3} & 1 & K_{4,5} & K_{4,6} \\ K_{5,1} & K_{5,2} & K_{5,3} & K_{5,4} & 1 & K_{5,6} \\ K_{6,1} & K_{6,2} & K_{6,3} & K_{6,4} & K_{6,5} & 1 \end{bmatrix}$$

Second-order interaction coeff. matrix

$$C_2 = \begin{bmatrix} K_{1,7} & K_{1,8} & K_{1,9} & K_{1,10} & \cdots\cdots & K_{1,27} \\ K_{2,7} & K_{2,8} & K_{2,9} & K_{2,10} & \cdots\cdots & K_{2,27} \\ K_{3,7} & K_{3,8} & K_{3,9} & K_{3,10} & \cdots\cdots & K_{3,37} \\ K_{4,7} & K_{4,8} & K_{4,9} & K_{4,10} & \cdots\cdots & K_{4,27} \\ K_{5,7} & K_{5,8} & K_{5,9} & K_{5,10} & \cdots\cdots & K_{5,27} \\ K_{6,7} & K_{6,8} & K_{6,9} & K_{6,10} & \cdots\cdots & K_{6,27} \end{bmatrix}$$

Equations (11)-(16) can then be written as,

$$L_u = C_1 L + C_2 P \tag{17}$$

or

$$\mu H = C_1 L + C_2 P \tag{18}$$

Now if $C_2 P$ is subtracted from each side and then each side is premultiplied by the inverse of C_1, Equation (18) becomes,

$$C_1^{-1} \mu H - C_1^{-1} C_2 P = C_1^{-1} C_1 L = L \tag{19}$$

or

$$L = C_1^{-1} \mu H - C_1^{-1} C_2 P \quad . \tag{20}$$

Letting $L_u \equiv \mu H$, the uncorrected loads which can always be calculated directly from the balance output channels, and

$$M \equiv C_1^{-1} C_2$$

then

$$L = C_1^{-1} L_u - MP \quad . \tag{21}$$

The values for C_1^{-1} and M are known from the balance calibration (see next section). The values for L_u can be calculated from the balance output channel readings and, for the first approximation, P can be calculated using the uncorrected loads, L_u; therefore, for the first iteration

$$L_1 = C_1^{-1} L_u - MP \quad . \tag{22}$$

where

$$P = (L_i)_u (L_j)_u \qquad \begin{array}{l} i = 1,2,\ldots 6 \\ j = i \ldots 6 \end{array} \tag{23}$$

For the second iteration use

$$P \equiv P_1 = (L_i)_1 (L_j)_1 \tag{24}$$

and

$$L_2 = C_1^{-1} L_u - MP_1 \quad . \tag{25}$$

The n^{th} iteration would then be

$$L_n = C_1^{-1} L_u - MP_{n-1} \quad . \tag{26}$$

These iterations should be continued until the differences in the interaction loads are equal to the resolution of the system, or

$$|MP_{n-1} - MP_{n-2}| \leq |\text{resolution}| \quad . \tag{27}$$

Generally the digital resolution of the recording system is expressed in counts, with one count being the smallest unit of resolution, and the sensitivity of the output channel, μ_i, is expressed as load/count. In that case the resolution is equal to the sensitivity μ_i. The following limit has proven to be adequate to obtain accurate results:

$$|MP_{n-1} - MP_{n-2}| \leq \frac{1}{2} \mu_i \tag{28}$$

Usually only two or three iterations are needed to obtain this limit.

Some of the computer routines used to perform the work reported on in this report are enclosed in Appendices A, B and C. Appendix C contains the data-reduction subroutines which are used to accomplish the above technique.

BALANCE CALIBRATION

The purpose of conducting a balance calibration is to measure the balance response to known loads in such a manner that the inverse procedure can be utilized to measure unknown loads as a function of the balance response to these loads. As mentioned earlier, a second-order, combined load calibration has always proved sufficient. This type of calibration is performed by first determining the balance response to pure primary loads and then, to combinations of these loads. This is accomplished by attaching a lightweight, rigid calibrating bar to the balance and then applying loads to the bar. The purpose of the bar is to provide a way of transferring loads to the balance in the same manner as they are transferred from the model to the balance. Primary loads are applied one at a time and the balance response on all channels is recorded. As an example, if pure normal forces are applied to the balance center by hanging weights at the center, all of the loads other than normal force drop out of Equations (11)-(16) and the remaining terms can be utilized to solve for the primary sensitivity of the balance to normal force.

For n weights there are n sets of the following equations:

$$F_N = \mu_1 \theta_1 - (K_{1,7} F_N^2) = \text{weight} \tag{29}$$

$$F_Y = \mu_2 \theta_2 - (K_{2,1} F_N + K_{2,7} F_N^2) = 0 \tag{30}$$

$$M_Y = \mu_3 \theta_3 - (K_{3,1} F_N + K_{3,7} F_N^2) = 0 \tag{31}$$

$$M_Z = \mu_4 \theta_4 - (K_{4,1} F_N + K_{4,7} F_N^2) = 0 \tag{32}$$

$$M_X = \mu_5 \theta_5 - (K_{5,1} F_N + K_{5,7} F_N^2) = 0 \tag{33}$$

$$F_A = \mu_6 \theta_6 - (K_{6,1} F_N + K_{6,7} F_N^2) = 0 \tag{34}$$

Equation (29) can be solved by fitting the values of μ_1 as a function of applied weight in order to determine μ_1 and $K_{1,7}$. The results expressed on the other channels as a function of normal force are recorded and saved for later reduction. Each of the other primary loads can be added individually and each primary sensitivity obtained in the same fashion. Once the primary sensitivities are known, it is possible to go back and reduce the data recorded on the other balance channels. For example, Equations (11)-(16) can now be fitted in order to obtain the remaining unknown coefficients, $K_{i,1}$ and $K_{i,7}$ ($i = 2 \rightarrow 6$). After this has been accomplished, all of the primary sensitivities and all of the first-order interaction coefficients are known.

In the determination of the second-order interaction coefficients it is necessary to apply combined loads to the calibrating bar. For example, if both a normal force and a yaw force are applied simultaneously, substitution of the loads into Equations (11)-(16) would give the following equations:

$$F_N = \mu_1\theta_1 - (K_{1,2}F_Y + K_{1,7}F_N^2 + K_{1,8}F_NF_Y + K_{1,13}F_Y^2) \tag{35}$$

$$F_Y = \mu_2\theta_2 - (K_{2,1}F_N + K_{2,7}F_N^2 + K_{2,8}F_NF_Y + K_{2,13}F_Y^2) \tag{36}$$

$$0 = \mu_3\theta_3 - (K_{3,1}F_N + K_{3,2}F_Y + K_{3,7}F_N^2 + K_{3,8}F_NF_Y + K_{3,13}F_Y^2) \tag{37}$$

$$\vdots \tag{38}$$

$$\vdots \tag{39}$$

$$0 = \mu_6\theta_6 - (K_{6,1}F_N + K_{6,2}F_Y + K_{6,7}F_N^2 + K_{6,8}F_NF_Y + K_{6,13}F_Y^2) \tag{40}$$

Since the terms $K_{i,8}$(i = 1 - 6) are the only unknown terms, they can be easily solved for as follows:

$$K_{i,8} = \frac{\mu_i\theta_i - K_{i,1}F_N - K_{i,2}F_Y - K_{i,7}F_N^2 - K_{i,13}F_Y^2}{F_NF_Y} \tag{41}$$

for i = 1 - 6.

In principle, only one combined load has to be loaded in order to solve for $K_{i,8}$, but in order to minimize errors and to help check the assumption that second order is the highest order of nonlinearity, several are loaded and the series are fit to obtain $K_{i,8}$(i = 1 - 6). The remaining second-order coefficients are obtained in a similar fashion.

In order to expedite the handling of the calibration data, the following definitions were made.

$\theta_{\ell,i,n}$ = balance output

$L_{\ell,t,n}$ = loads applied

$K_{i,\ell}$ = balance coefficients

where

- ℓ = group number (1-27) the same as used to define the 27-load configurations on page 12.
- i = output channel number (1-6)
- n = sequential numerical order of the weights (1-15)
- t = type of load

The correspondence between these values is as follows:

Load	Type, t
F_N	1
F_Y	2
M_Y	3
M_Z	4
M_X	5
F_A	6

	LOAD TYPE		
Load Group	Primary	Secondary	Coefficient
1	1		$K_{i,1}$
2	2		$K_{i,2}$
3	3		$K_{i,3}$
4	4		$K_{i,4}$
5	5		$K_{i,5}$
6	6		$K_{i,6}$
7	1	1	$K_{i,7}$
8	1	2	$K_{i,8}$
9	1	3	$K_{i,9}$
10	1	4	$K_{i,10}$
11	1	5	$K_{i,11}$
12	1	6	$K_{i,12}$
13	2	2	$K_{i,13}$
14	2	3	$K_{i,14}$
15	2	4	$K_{i,15}$
16	2	5	$K_{i,16}$
17	2	6	$K_{i,17}$

LOAD TYPE (Cont'd)

Load Group	Primary	Secondary	Coefficient
18	3	3	$K_{i,18}$
19	3	4	$K_{i,19}$
20	3	5	$K_{i,20}$
21	3	6	$K_{i,21}$
22	4	4	$K_{i,22}$
23	4	5	$K_{i,23}$
24	4	6	$K_{i,24}$
25	5	5	$K_{i,25}$
26	5	6	$K_{i,26}$
27	6	6	$K_{i,27}$

The equations for the primary and first-order interaction coefficients, Equations (29)-(34), can then be written as,

$$\theta_{\ell,i,n} = A_{i,\ell}L_{\ell,\ell,n} + B_{i,I}L^2_{\ell,\ell,n} \tag{42}$$

where for $\ell = 1 - 6$,

$$i = 1 - 6$$

$$I = 21 - (6 - \ell)\left(\frac{7 - \ell}{2}\right) + \ell$$

$$A_{i,\ell} = K_{i,\ell}/\mu_i$$

$$B_{i,I} = K_{i,I}/\mu_i$$

For each of the six load setups Equation (42) can be fitted for the six channels and n data points in order to obtain the 72 values for $A_{i,\ell}$ and $B_{i,I}$. For the six sets of $A_{i,\ell}$ and $B_{i,I}$ where $i = \ell$ the values of μ_i can be found since for $i = \ell$, $K_{i,\ell} = 1$ and $\mu_i = 1/A_{i,\ell}$. The first-order interactions can then be calculated from,

$$K_{i,\ell} = \mu_i A_{i,\ell} \tag{43}$$

$$K_{i,I} = \mu_i B_{i,I} \quad . \tag{44}$$

The equations for the second-order interactions, Equation (41), can be expressed as follows.

$$K_{i,\ell} = \frac{\mu_i \theta_{\ell,i,n} - K_{i,A} L_{\ell,A,n} - K_{i,B} L_{\ell,B,n} - K_{i,C} L^2_{\ell,A,n} - K_{i,D} L^2_{\ell,B,n}}{L_{\ell,A,n} L_{\ell,B,n}} \tag{45}$$

for i = 1 - 6 and where

ℓ	A	B
8-12	1	ℓ - 6
14-17	2	ℓ - 11
19-21	3	ℓ - 15
23,24	4	ℓ - 18
26	5	ℓ - 20

$$C = 21 - (6 - A)\left(\frac{7 - A}{2}\right) + A$$

$$D = 21 - (6 - B)\left(\frac{7 - B}{2}\right) + B$$

The main routines used in the balance calibration program have been listed in Appendix B.

MODEL WEIGHT EFFECTS

Since a combined load, second-order calibration is used to calibrate the balance and since this necessitates referencing all loads to a condition of zero load-zero balance reading, the act of just placing a model on the balance will be indicated as a load in the balance readings. With a six-component balance, these balance readings can be used to determine the model weight and center of gravity. These values could then be used to calculate the balance load caused by the model as the model orientation is changed in the tunnel. The subtraction of these loads from the loads measured during the run would then leave the desired aerodynamic load.

The above scheme for accounting for model weight effects on the balance has several shortcomings. First, it only accounts for static effects. Dependent upon the model mass and pitch history, there may be significant dynamic loading effects that would have to be corrected for either experimentally or analytically. Secondly, a six-component balance must be used. If only a four-component balance is used and the model center of gravity is not on the balance axes, it is not possible to accurately compensate for model load effects. Another problem that arises in calculating model weight effects is that the balance mass also affects the readings. If the effects of model mass on the balance readings are to be calculated as a function of model orientation, it is essential (in some cases) to account for both the model mass and the balance mass and their different centers of gravity independently. Furthermore, the apparent balance mass that influences the gage readings is different for different gage sections.

Another scheme for accounting for model and balance mass effects is to sweep the model through the same orientations that will be used in the data run. During this sweep, record the balance output channels, reduce these data to loads as a function of model orientation, and then subtract these measured loads from the data acquired during the test. The disadvantage in this technique arises if relatively large sting and balance deflections occur as a result of the aerodynamic loads. If that occurs the mass effects may not be known for the same exact orientation that occurs during the test.

Neither of the above schemes is ideal. Dependent on the test parameters, such as balance mass/model mass, aerodynamic load/model weight, pitch angles, pitch rates, static deflections, etc., either of the techniques may be preferred. If sting deflections are minimal, the technique of pitching the model and recording the mass effects is generally preferred at the White Oak Laboratory.

TARE MEASUREMENTS

Tare measurements are sample recordings of all test parameters made under a no-flow condition. These recordings are made so that a zero reference point can be established for all instrumentation just prior to the acquisition of data. This is done by recording a short sample of the balance data acquired at either a fixed pitch angle or during a sweep. These readings are then compared to the readings acquired during the weight effects sweep for the same angular orientation. The differences between the balance tare readings and the weight effect readings are then stored as balance offsets which are applied to all of the balance data recorded immediately after the tare.

If any change in the balance output channels occurs due to thermal effects, it is quite important to obtain a new tare in order to update the balance reference reading.

DATA REDUCTION

In the data reduction each line of data (balance data recorded at one point in time) is reduced independently from the others. The first record of all balance channels and angular positions is read from the data tape. These data are then adjusted by subtracting the balance offsets that were determined in the tare. These adjusted readings are then reduced to loads through the iterative process described in the section on General Load Resolution Technique. The aerodynamic loads are then found by first interpolating the weight loads in order to get the proper model effect at the same orientation for which the data were recorded and then subtracting the model load from the measured load.

Once the loads are known, it is possible to calculate how much the balance has been deflected by these loads. The deflections are calculated from the information determined in the static angular calibration and added to the measured orientation in order to determine the true orientation.

These loads and orientations are then reduced to coefficient form in the desired axis system in a separate subroutine called COEFF. Generally the coefficients are defined in the balance axes as follows:

$$AFC = C_a = \frac{-F_X}{QA}$$

$$YFC = C_y = \frac{F_Y}{QA}$$

$$NFC = C_N = \frac{-F_Z}{QA}$$

$$RMC = C_\ell = \frac{M_X}{QA\ell}$$

$$PMC = C_m = \frac{M_Y}{QA\ell}$$

$$YMC = C_n = \frac{M_Z}{QA\ell}$$

where

Q = dynamic pressure (psi)

A = reference area (inches^2)

ℓ = reference length (inches)

$-F_X = F_A$ = axial force (lb)

F_Y = yaw force (lb)

$-F_Z = F_N$ = normal force (lb)

M_X = roll moment (in-lb)

M_Y = pitch moment (in-lb)

M_Z = yaw moment (in-lb)

The various axis systems and transformations are shown in Appendix A and the data-reduction subroutines are shown in Appendix C.

GENERAL COMMENTS

The purpose of this report is to explain the concepts utilized at the White Oak Laboratory in making static force and moment measurements. In the actual utilization of these techniques many problems arise that are rarely ever brought out in a general discussion, and unfortunately, it is these problems and how they are handled that determine the quality of the data obtained. Some of these concerns, such as the accounting for model mass effects and balance drifts, have been alluded to already.

In general, there are four areas that must be examined and checked in order to obtain quality data.

The first question is, What is the quality of flow? Parameters of concern here are the uniformity of the flow over the model during the test period. These qualities are tunnel dependent and outside the scope of this report but are still of primary importance to the value of the data.

The second area of concern is the establishment of reference conditions. Included in these are the details involved in determining the balance output with no aerodynamic load and in determining the calibration constants. It is imperative that check loads (multiple combined loads) be placed on the model and/or balance and reduced in order to check the validity of the calibrations and the accuracy of pure static measurements without the complications of flow.

The third area of concern is the establishment of the alignment of the model and balance with respect to the flow. This is especially true for nonsymmetric models for which the normal force and pitching moment are not zero at zero degree angle of attack. With a symmetric model it is possible to move the model in the flow until the orientation is determined for which no aerodynamic force or moment exists. This location establishes the point where the total angle of attack is zero. Depending upon the accuracy desired and the facility in which the test is being conducted, it is sometimes necessary to utilize a symmetric model to establish these conditions as reference conditions for a nonsymmetric model. For models which have a slight asymmetry, it is possible to test them at various roll angles (for example, 0, ±90, 180 degrees) in order to find the orientation of the model with respect to the flow.

The fourth area that must be analyzed carefully is not always present but certainly must be reckoned with in some small hot facilities. This is the problem of how to handle balance drifts and shifts which are usually caused by thermal gradients across the balance sections. If it is not possible to insulate or cool the balance it is usually necessary to limit the duration in which the model is exposed to the hot flow. This may mean that it is necessary to obtain data for just a few fixed angles rather than to leave the model in the flow and sweep through a large range of angles. If a test is being conducted by sweeping the model through large angles of attack, the thermal effects are angle dependent and may not be evident to the project engineer unless multiple sweeps are made and the data are compared. It is possible for the thermal effects to follow the angle of attack closely enough so that the differences between tares conducted before and after the run are very small but yet the effect on the data at high angles may be quite large.

The ideal way to eliminate the above-mentioned problems would be to improve the design of the balances and tunnels, but since this is not always financially possible, the project engineer must resort to procedural changes in the manner in which the test is conducted. This may mean that additional flow alignment runs are necessary, or that sweeps may not be possible, or that additional tares may have to be taken. All of these things must be considered in the initial design of the test program.

APPENDIX A

AXIS TRANSFORMATIONS

Within the data-reduction process there are a number of axis systems which are used. The tunnel axes are considered to be the reference axes. These axes may be defined slightly different in each of the tunnels but in general both the X_T and Y_T axes are defined to be in a horizontal plane (established with a precision level) with the X_T axis pointing into the flow and the vertical axis, Z_T, pointing down. All angular rotations of the sting are defined with respect to the tunnel axes through the angles θ, ψ, and ϕ, and generally, in that order. Since the tunnel axes are somewhat arbitrary, the matter of real importance is the location of the sting with respect to the wind vector. The small angles $\Delta\theta_W$ and $\Delta\psi_W$ are used to define the tunnel axes with respect to the wind axes (See Fig. A-1).

The transformation matrix to go from the wind axes to the tunnel axes in the order of pitch, yaw, and roll is called $[\ell_{WT}]$.

$$[\ell_{WT}] = \begin{bmatrix} \cos\Delta\psi_W\cos\Delta\theta_W & \sin\Delta\psi_W & -\cos\Delta\psi_W\sin\Delta\theta_W \\ -\sin\Delta\psi_W\cos\Delta\theta_W & \cos\Delta\psi_W & \sin\Delta\psi_W\sin\Delta\theta_W \\ \sin\Delta\theta_W & 0 & \cos\Delta\theta_W \end{bmatrix}$$

where the order is $\Delta\theta_W \rightarrow \Delta\psi_W \rightarrow \Delta\phi_W$.

The transformation matrix to go from the tunnel axes to the sting axes is called $[\ell_{TS}]$.

$$[\ell_{TS}] = \begin{bmatrix} \cos\psi\cos\theta & \sin\psi & -\cos\psi\sin\theta \\ -\cos\phi\sin\psi\cos\theta + \sin\phi\sin\theta & \cos\phi\cos\psi & \cos\phi\sin\psi\sin\theta + \sin\phi\cos\theta \\ \sin\phi\sin\psi\cos\theta + \cos\phi\sin\theta & -\sin\phi\cos\psi & -\sin\phi\sin\psi\sin\theta + \cos\phi\cos\theta \end{bmatrix}$$

where the order is $\theta \rightarrow \psi \rightarrow \phi$.

If a dogleg sting is used or if the first rotation is in the yaw plane, a different transformation matrix must be used to go from the tunnel axes to the sting axes. The sting orientation would be as shown in Figure A-2. The transformation matrix would then be,

$$[\ell_{TSY}] = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ -\cos\phi\sin\psi + \sin\phi\sin\theta\cos\psi & \cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi & \sin\phi\cos\theta \\ \sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi & -\sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi & \cos\theta\cos\phi \end{bmatrix}$$

where the order is $\psi \rightarrow \theta \rightarrow \phi$.

The variation in the balance orientation with respect to the sting is due to the deflection of the balance and sting due to the loads. These deflections are defined as δ, γ, and ε (see Fig. A-3). The transformation matrix is $[\ell_{SB}]$.

$$[\ell_{SB}] = \begin{bmatrix} \cos\gamma\cos\delta & \sin\gamma & -\cos\gamma\sin\delta \\ -\cos\varepsilon\sin\gamma\cos\delta + \sin\varepsilon\sin\delta & \cos\varepsilon\cos\gamma & \cos\varepsilon\sin\gamma\sin\delta + \sin\varepsilon\cos\delta \\ \sin\varepsilon\sin\gamma\cos\delta + \cos\varepsilon\sin\delta & -\sin\varepsilon\cos\gamma & -\sin\varepsilon\sin\gamma\sin\delta + \cos\varepsilon\cos\delta \end{bmatrix}$$

where the order is $\delta \rightarrow \gamma \rightarrow \varepsilon$.

If the model is rolled with respect to the balance through an angle, ϕ_M, the transformation matrix is $[\ell_{BM}]$ (see Fig. A-3).

$$[\ell_{BM}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_M & \sin\phi_M \\ 0 & -\sin\phi_M & \cos\phi_M \end{bmatrix}$$

When the model moment reference center is not located at the balance center, it is necessary to translate the balance moments. The distances along the rotated axes are called DXMC, DYMC, and DZMC and are positive if the moment reference center is located forward, right, or down from the balance center (see Fig. A-4). The moments referenced to the model moment reference center are calculated as follows.

$$\begin{bmatrix} M_{XC} \\ M_{YC} \\ M_{ZC} \end{bmatrix} = \begin{bmatrix} M_{X'} \\ M_{Y'} \\ M_{Z'} \end{bmatrix} + \begin{bmatrix} DXMC \\ DYMC \\ DZMC \end{bmatrix} \begin{bmatrix} 0 & F_N' & F_Y' \\ -F_N' & 0 & F_A' \\ -F_Y' & -F_A' & 0 \end{bmatrix}$$

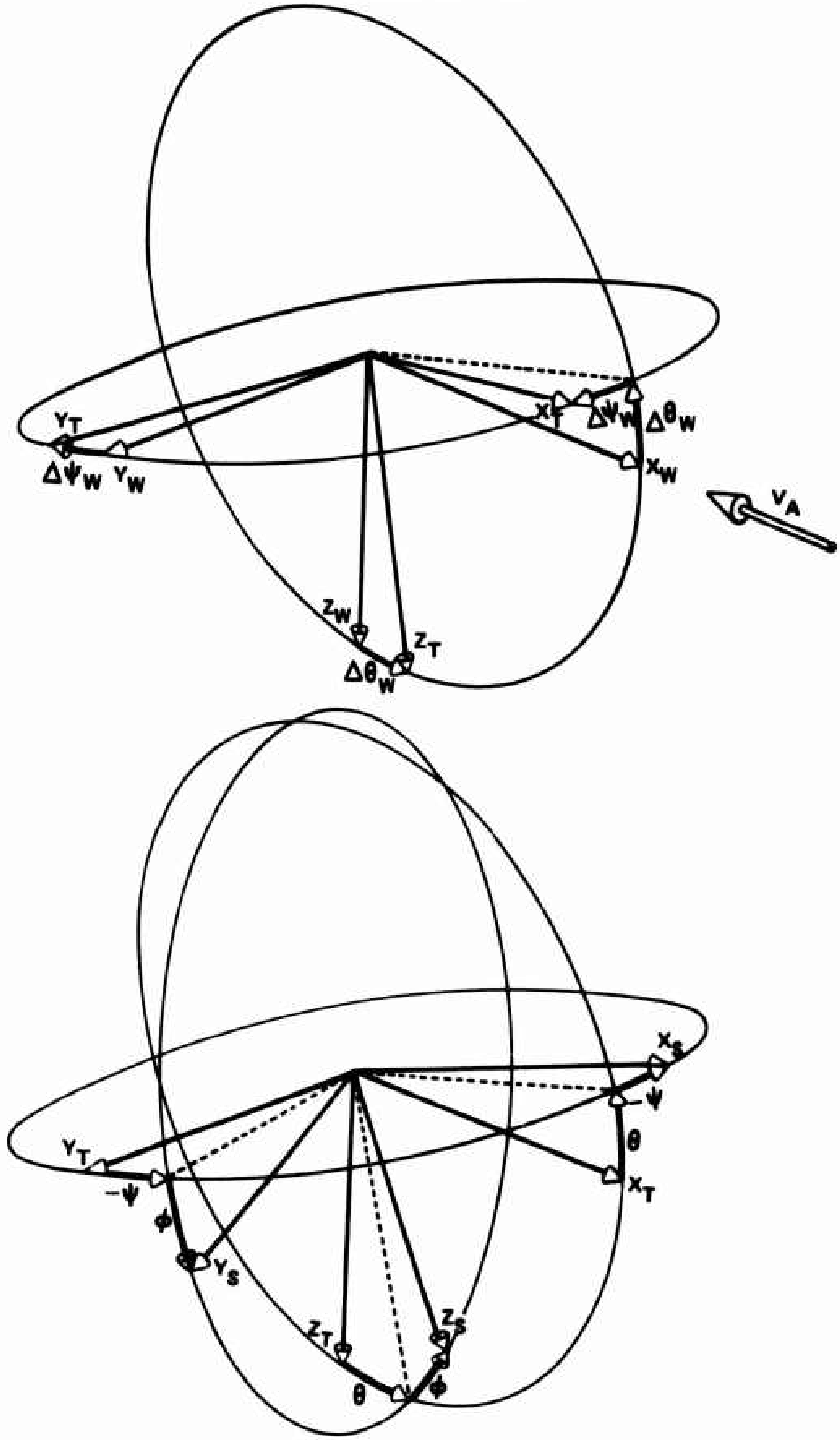

FIG. A-1 TUNNEL, STING, AND WIND AXES

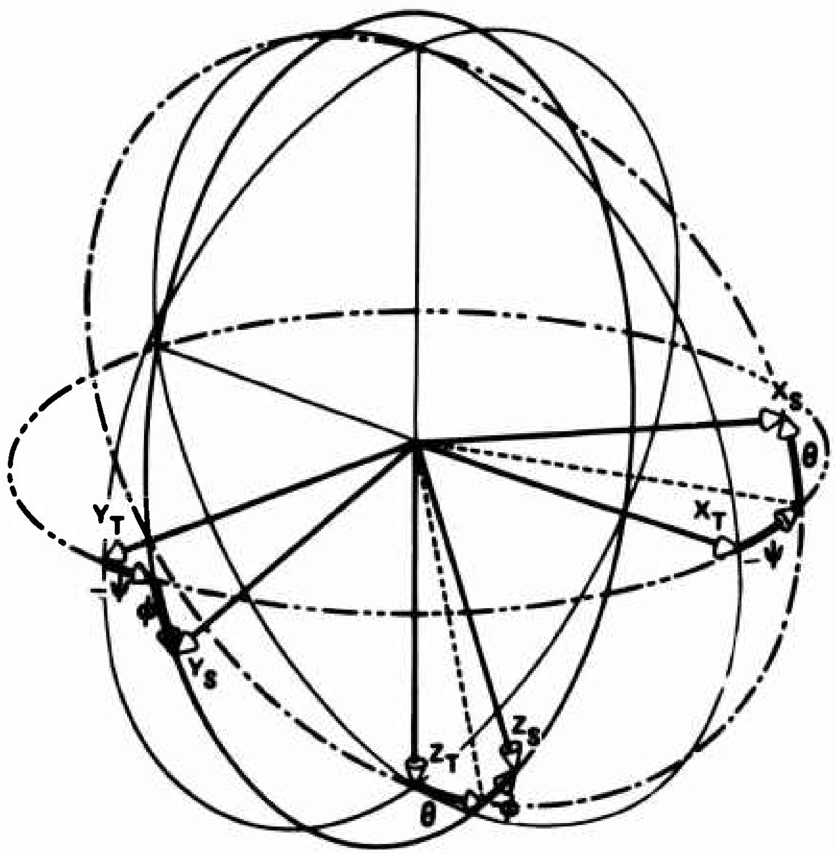

FIG. A-2 TUNNEL AND STING AXES FOR A YAW, PITCH, ROLL SEQUENCE

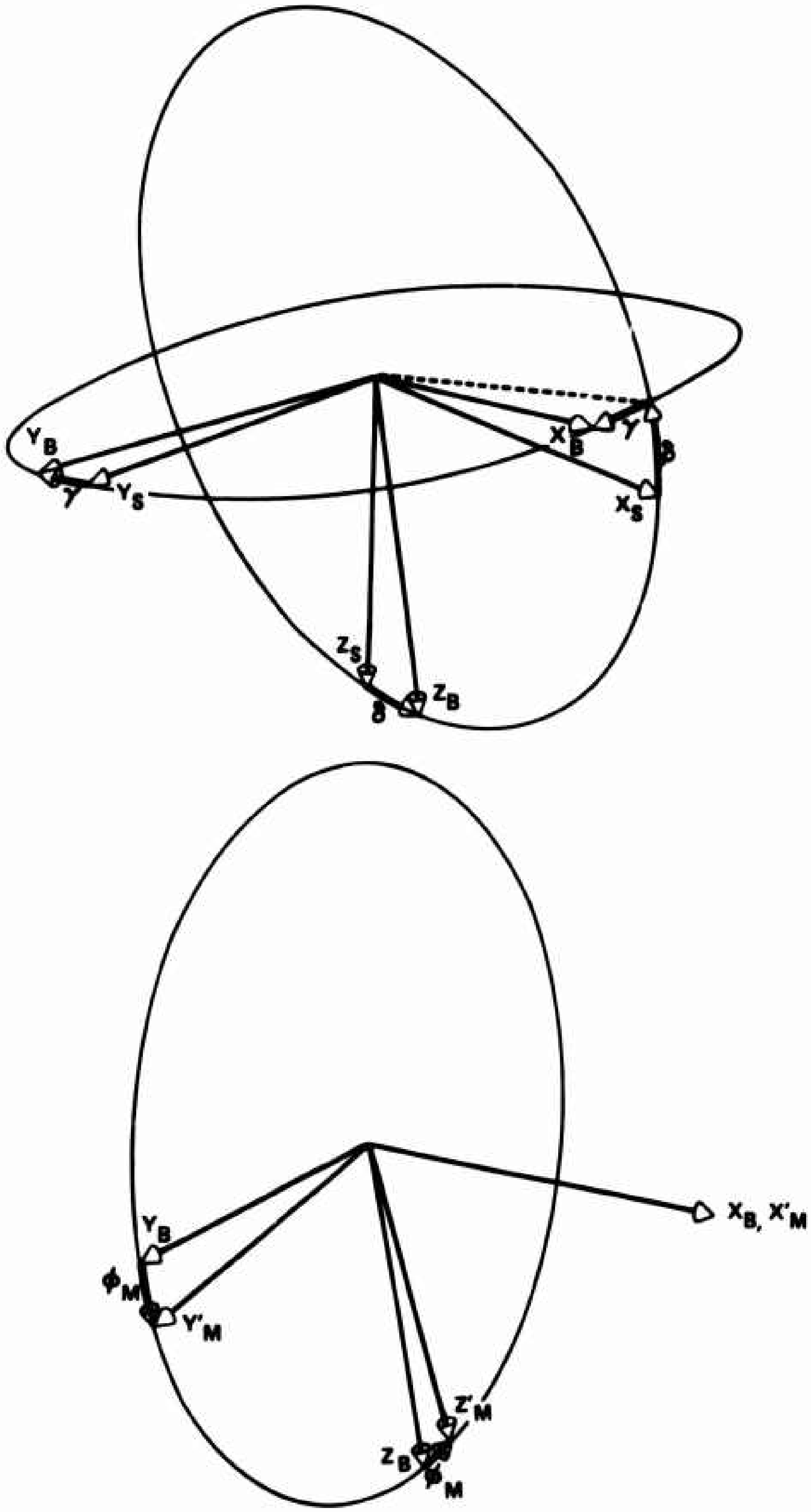

FIG. A-3 STING, BALANCE, AND MODEL AXES

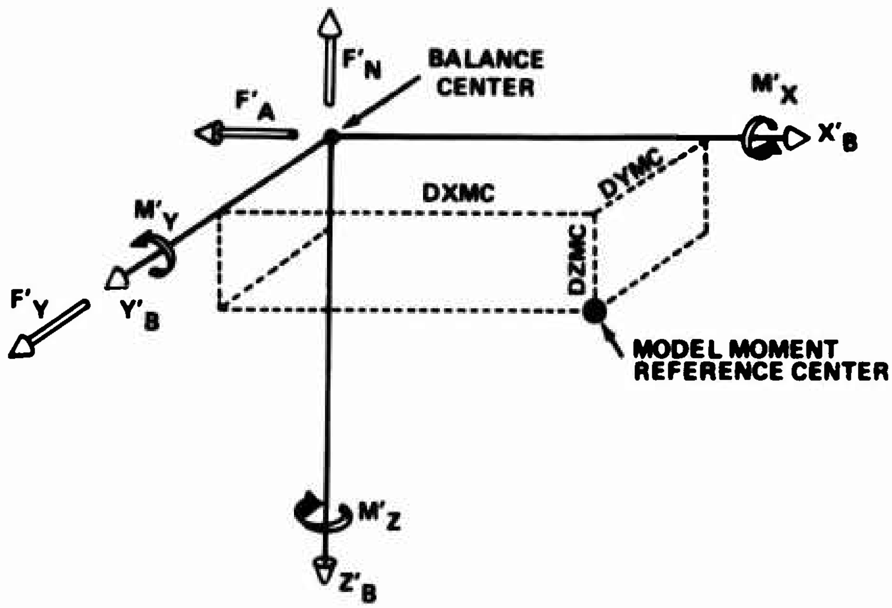

FIG. A-4 RELATIONSHIP BETWEEN BALANCE CENTER AND MODEL MOMENT REFERENCE CENTER

APPENDIX B

BALANCE CALIBRATION PROGRAM

BALANCE CALIBRATION PROGRAM
APPENDIX B

```
      PROGRAM BALCAL(INPUT=64,OUTPUT    =64,TAPE5=INPUT,TAPE6=OUTPUT)     BAL00100
C
C  THIS PROGRAM IS USED TO REDUCE BALANCE CALIBRATION LOADINGS
C
C  THE FOLLOWING INPUTS ARE DEFINED AS.......
C         BAL = NAME OF BALANCE
C         MC = NUMBER OF BALANCE COMPONENTS
C         ITYPE = 1 INDICATES A MOMENT BALANCE
C               =  2 INDICATES A FORCE-MOMENT BALANCE
C         DATE = DATE CALIBRATION WAS PERFORMED
C         ENG = ENGINEER PERFORMING CALIBRATION
C         DARE = DATA ACQUISITION SYSTEM USED
C         SETSCAL = NOMINAL MV FULL SCALE SENSITIVITY....NOTE IF THIS
C         IS A REDUCTION OF A MOMENT BALANCE, ITYPE = 1, THE SETSCAL
C         VALUES FOR CHANNEL 1 MUST EQUAL THE VALUES FOR CHANNEL 2
C         AND CHANNEL 3 MUST MATCH CHANNEL 4.
C         OFF = INITIAL READINGS OF ALL CHANNELS WITH NO LOAD
C         SCALE = SENSITIVITY SETTINGS AT WHICH LOADS WERE MEASURED
C         WT = VALUES OF LOADS HUNG (LB OR IN-LB)
C         R = DARE CHANNEL READINGS (COUNTS)
C         L = LOAD TYPE NUMBER
C         M = CHANNEL NUMBER
C         N = LOAD NUMBER
C         LS = 11 AT END OF INDIVIDUAL LOADING SETUP
C          =    99 AT END OF CALIBRATION
C
      DIMENSION WT(27,6,10),A(6,27),B(6,27),BC(6,27),X(10),Y(10),AN(2), BAL00110
     1PRMSEN(6),OFF(6),RD(27,6,10),SCALE(6,27),R(27,6,10),TITLE(27),
     1FL(6),PL(36),C1L(6),C2P(6),THCK(6),DELT(6),NN(27),SETSCAL(6)
      DIMENSION BAL(2),DATE(2),ENG(2)
      REAL NH(6)
      DATA TITLE/2HFN,2HFY,2HMY,2HMZ,2HMX,2HFA,4HFNFN,4HFNFY,4HFNMY,     BAL00150
     14HFNMZ,4HFNMX,4HFNFA,4HFYFY,4HFYMY,4HFYMZ,4HFYMX,4HFYFA,4HMYMY,     BAL00160
     14HMYMZ,4HMYMX,4HMYFA,4HMZMZ,4HMZMX,4HMZFA,4HMXMX,4HMXFA,4HFAFA/     BAL00170
*                                                                         BAL00180
*  ARRAYS ARE INITIALIZED                                                 BAL00190
*                                                                         BAL00200
      DO 8 M=1,6
    8 OFF(M)=THCK(M)=DELT(M)=PRMSEN(M)=0.0
      DO 9 L=1,27                                                         BAL00230
      NN(L)=0                                                             BAL00240
      DO 9 M=1,6                                                          BAL00250
      A(M,L)=0.0                                                          BAL00260
      SCALE(M,L)=0.0
      BC(M,L)=0.0                                                         BAL00270
      DO 9 N=1,10                                                         BAL00280
    9 WT(L,M,N)=0.                                                        BAL00290
*                                                                         BAL00300
*  ALL CALIBRATION DATA ARE READ IN HERE                                  BAL00310
*                                                                         BAL00320
   10 READ(5,8002) BAL,MC,ITYPE
      READ(5,8002) DATE                                                   BAL00340
      READ(5,8002) ENG                                                    BAL00350
      READ(5,8002) DARE
      READ(5,8005) DUM,DUM,(SETSCAL(M),M=1,6)
 8002 FORMAT(2A10,I5,I5)
   20 READ(5,8005) L,DUM,(OFF(M),M=1,6)
   30 READ(5,8005) L, DUM,(SCALE(M,L),M=1,6)
      DO 35 M=1,MC
   35 OFF(M)=OFF(M)*SCALE(M,L)/SETSCAL(M)
   40 READ(5,8005) L,N,(WT(L,M,N),M=1,6),LS                               BAL00390
```

APPENDIX B (CONT.)

```
 8005 FORMAT(2I5,6F10.0,I2)                                              BAL00410
*                                                                        BAL00420
*        LS = 11 AT END OF INDIVIDUAL LOADING SETUPS
*        LS = 99 AT END OF CALIB DATA                                    BAL00440
*                                                                        BAL00470
   50 READ(5,8005) L,N,(H(L,M,N),M=1,6),LS                               BAL00480
*
*  READINGS ARE NOW ADJUSTED FOR PROPER SCALE SETTINGS
*
   52 DO 55 M=1,MC
      NH(M)=H(L,M,N)
      NH(M)=NH(M)         *SCALE(M,L)/SETSCAL(M)
   55 CONTINUE
*                                                                        BAL00490
*  MOMENT GAGE READINGS ARE ADJUSTED FOR UTILIZATION AS FORCE            BAL00500
*  AND MOMENT GAGES IF NECESSARY, AND ADJUSTMENTS ARE MADE FOR OFFSETS
*                                                                        BAL00520
      IF(ITYPE.EQ.1) GO TO 60
      GO TO 62
   60 RD(L,1,N)=NH(2)-OFF(2)-(NH(1)-OFF(1))
      RD(L,2,N)=NH(4)-OFF(4)-(NH(3)-OFF(3))
      RD(L,3,N)=NH(1)-OFF(1)+NH(2)-OFF(2)
      RD(L,4,N)=NH(3)-OFF(3)+NH(4)-OFF(4)
      RD(L,5,N)=NH(5)-OFF(5)
      RD(L,6,N)=NH(6)-OFF(6)
      GO TO 70
   62 DO 65 M=1,MC
   65 RD(L,M,N)=NH(M)-OFF(M)
   70 NN(L)=N                                                            BAL00590
      IF(LS.EQ.11)  GO TO 20                                             BAL00594
      IF(LS.EQ.99)  GO TO 80                                             BAL00596
      GO TO 40                                                           BAL00600
*                                                                        BAL00610
*  SOLUTION FOR PRIMARY SENSITIVITIES, FIRST ORDER INTERACTIONS,         BAL00620
*  AND SECOND ORDER LOAD SQUARED INTERACTIONS                            BAL00630
*                                                                        BAL00640
   80 CONTINUE                                                           BAL00650
      GO TO 400                                                          BAL00652
 90   CONTINUE                                                           BAL00654
      DO 200 L=1,MC                                                      BAL00660
      DO 190 I=1,MC                                                      BAL00670
      NO=NN(L)                                                           BAL00680
      IF(NO.EQ.0)  GO TO 200                                             BAL00690
      DO 150 N=1,NO                                                      BAL00700
      X(N)=WT(L,L,N)                                                     BAL00710
      Y(N)=RD(L,I,N)                                                     BAL00720
  150 CONTINUE                                                           BAL00730
*                                                                        BAL00740
*        EQUATION   Y=A(1)*X+A(2)*X**2                                   BAL00750
*                                                                        BAL00760
      CALL FIT1(NO,X,Y,AN)                                               BAL00770
*                                                                        BAL00780
      A(I,L)=AN(1)                                                       BAL00790
      II=21-((6-L)*(7-L)/2)+L                                            BAL00800
      B(I,II)=AN(2)                                                      BAL00810
  190 CONTINUE                                                           BAL00820
  200 CONTINUE                                                           BAL00830
*                                                                        BAL00840
*        PRIMARY SENSITIVITIES                                           BAL00850
*                                                                        BAL00860
      DO 210 I=1,MC                                                      BAL00870
      BC(I,I)=1.                                                         BAL00880
```

APPENDIX B (CONT.)

```
      IF(A(I,I).EQ.0.0)  GO TO 210
      PRMSEN(I)=1./A(I,I)
  210 CONTINUE
*
*        FIRST ORDER INTERACTIONS
*
      DO 230 L=1,MC
      DO 229 I=1,MC
      IF(I.EQ.L) GO TO 227
      BC(I,L)=PRMSEN(I)*A(I,L)
*
*        SECOND ORDER LOAD SQUARED INTERACTIONS
*
227   II=21-((6-L)*(7-L)/2)+L
      BC(I,II)=PRMSEN(I)*A(I,II)
  229 CONTINUE
  230 CONTINUE

*  SOLUTION OF REMAINING SECOND ORDER INTERACTIONS

      DO 300 L=8,26
      NO=NN(L)
      IF(NO.EQ.0)  GO TO 300
      IF(L.GE.8.AND.L.LE.12) 235,237
  235 LA=1
      LB=L-6
      GO TO 270
  237 IF(L.EQ.13) 300,241
  241 IF(L.GE.14.AND.L.LE.17) 243,245
  243 LA=2
      LB=L-11
      GO TO 270
  245 IF(L.EQ.18) 300,247
  247 IF(L.GE.19.AND.L.LE.21) 249,251
  249 LA=3
      LB=L-15
      GO TO 270
  251 IF(L.EQ.22) 300,253
  253 IF(L.GE.23.AND.L.LE.24) 255,257
  255 LA=4
      LB=L-18
      GO TO 270
  257 IF(L.EQ.25) 300,259
  259 LA=5
      LB=L-20

  270 LC=21-((6-LA)*(7-LA)/2)+LA
      LD=21-((6-LB)*(7-LB)/2)+LB
      DO 290 I=1,MC
      DO 280 N=1,NO
      Y(N)=PRMSEN(I)*RD(L,I,N)-BC(I, LA)*WT(L,LA,N)-BC(I,LB)*WT(L,LB,N)-
     1 BC(I,LC)*WT(L,LA,N)**2-BC(I,LD)*WT(L,LB,N)**2
  280 X(N)=WT(L,LA,N)*WT(L,LB,N)
*
      CALL FIT2(NO,X,Y,AN)
*
*        EQUATION    Y=A(1)*X
*
  290 BC(I,L)=AN(1)
  300 CONTINUE

*  CALIBRATION LOADS AND READINGS LISTED
      GO TO 552
```

```
                                                                        BAL01510
  400 WRITE(6,9001)                                                     BAL01520
 9001 FORMAT(1H1,53X*NAVAL SURFACE WEAPONS CENTER*//57X*WHITE OAK LABORABAL01530
     1TORY*)                                                            BAL01540
      WRITE(6,9002) BAL,ENG                                             BAL01550
 9002 FORMAT(/4X*BALANCE   *2A10,70X*ENGINEER   *2A10)                  BAL01560
      WRITE(6,9003) DATE,DARE
 9003 FORMAT(4X*CALIBRATION DATE   *2A10,66X*DARE   *2A10)
      WRITE(6,9040) SETSCAL
 9040 FORMAT(/52X*MV FULL SCALE SETTINGS*6(F10.2)/)
      WRITE(6,9004)(TITLE(L),L=1,6),(TITLE(L),L=1,6)                    BAL01590
 9004 FORMAT(/*    L    N*6(8XA2),6X,6(8XA2))                           BAL01600
      WRITE(6,9005)                                                     BAL01610
 9005 FORMAT(10X,6(6X*LOAD*),5X,6(7X*RDG*))                             BAL01620
                                                                        BAL01630
      DO 500 L=1,27                                                     BAL01640
      NO=NN(L)                                                          BAL01650
      IF(NO.EQ.0)  GO TO 500                                            BAL01660
      DO 499 N=1,NO                                                     BAL01670
      WRITE(6,9006) L,N,(WT(L,M,N),M=1,6),(RD(L,M,N),M=1,6)             BAL01680
 9006 FORMAT(/2I4,4X,6F10.4,3X,6F10.1)                                  BAL01690
  499 CONTINUE                                                          BAL01700
  500 CONTINUE                                                          BAL01710
      WRITE(6,9001)
      WRITE(6,9002)  BAL,ENG
      WRITE(6,9003)  DATE,DARE
      WRITE(6,9007)(TITLE(L),L=1,6),(L,L=1,6)                           BAL01740
 9007 FORMAT(/*    L    N*6(8XA2),6X,6(8X*R*I1))                        BAL01750
      DO 550 L=1,27                                                     BAL01760
      NC=NN(L)                                                          BAL01770
      IF(NC.EQ.0)  GO TO 550                                            BAL01780
      WRITE(6,9040)  (SCALE(M,L), M=1,MC)
      DO 549 N=1,NO                                                     BAL01790
      WRITE(6,9006) L,N,(WT(L,M,N),M=1,6),(H(L,M,N),M=1,6)              BAL01800
  549 CONTINUE                                                          BAL01810
  550 CONTINUE                                                          BAL01820
                                                                        BAL01830
      GO TO 90                                                          BAL01837
552   CONTINUE                                                          BAL01838

*  CALIBRATION CONSTANTS LISTED                                         BAL01840
                                                                        BAL01850
      WRITE(6,9001)
      WRITE(6,9002)  BAL,ENG
      WRITE(6,9003)  DATE,DARE
      WRITE(6,9008)                                                     BAL01860
 9008 FORMAT(/   ,13X*CALIBRATION SYSTEM VALUES*30X*NORMALIZED INTERACTIO
     1N*/74X*COEFFICIENTS*)
      WRITE(6,9009)(NK,NK=1,6)                                          BAL01890
 9009 FORMAT(/11X*MOMENT*4X*DARE SCALE*26X,6I12)
      WRITE(6,9010)(TITLE(NK),NK=1,6)                                   BAL01910
 9010 FORMAT(12X*GAGE*5X*(MV-FULL SCALE)*22X,6(10XA2)/)
      DO 310 NK=1,6                                                     BAL01930
      WRITE(6,9011) NK,SETSCAL(NK)     ,NK,TITLE(NK),(HC(I,NK),I=1,6)   BA
 9011 FORMAT(14X,I1,3XF8.2,17X,I11,1XA2,3X,6E12.5)
  310 CONTINUE                                                          BAL01960
      DO 315 NK=7,10                                                    BAL01970
      WRITE(6,9017)NK,TITLE(NK),(HC(I,NK),I=1,6)                        BAL01980
 9017 FORMAT(52X,I2,1X,A4,1X,6E12.5)                                    BAL01990
  315 CONTINUE                                                          BAL02000
      WRITE(6,9021)  (HC(I,11),I=1,6)                                   BAL02010
 9021 FORMAT(12X    *PRIME SENSITIVITIES*21X*11 FNMX*1X,6E12.5)
      WRITE(6,9022)  (HC(I,12),I=1,6)                                   BAL02030
```

APPENDIX B (CONT.)

```
 9022 FORMAT(10X*(LB CR IN-LB PER COUNT)*19X*12 FNFA*1X,6F12.5)
      WRITE(6,9023) (BC(I,13),I=1,6)                                    BAL02050
 9023 FORMAT(52X*13 FYFY*1X,6E12.5)                                     BAL02060
      DO 330 NK=1,6                                                     BAL02070
      NZ=NK+13                                                          BAL02080
      WRITE(6,9024) TITLE(NK),PRMSEN(NK),NZ,TITLE(NZ),(BC(I,NZ),I=1,6)  BAL02090
 9024 FORMAT(12X,A2,5X,E12.5,21X,I2,1XA4,1X,6E12.5)                     BAL02100
  330 CONTINUE                                                          BAL02110
      DO 340 NK=20,27                                                   BAL02120
      WRITE(6,9030) NK,TITLE(NK),(BC(I,NK),I=1,6)                       BAL02130
 9030 FORMAT(52X,I2,1X,A4,1X,6E12.5)                                    BAL02140
  340 CONTINUE                                                          BAL02150
                                                                        BAL02160
*  CALIBRATION CHECK                                                    BAL02170
                                                                        BAL02180
      WRITE(6,9001)
      WRITE(6,9002)  BAL,ENG
      WRITE(6,9003)  DATE,DARE
      WRITE(6,9038)                                                     BAL02190
 9038 FORMAT(/  ,60X*CALIBRATION CHECK*//                               BAL02200
     134X*APPLIED WEIGHTS*34X*DIFFERENCE BETWEEN LOAD READING AND CALCUL
     2ATED*/84X*READING, K(6X27) X L(27X1) IN COUNTS*)
      WRITE(6,9041) SETSCAL
 9041 FORMAT(/55X*(MV-FULL SCALE)*6(F10.2))
      WRITE(6,9004) (TITLE(L),L=1,6),(TITLE(L),L=1,6)                   BAL02230
      WRITE(6,9005)                                                     BAL02240
      DO 1000 L=1,27                                                    BAL02250
      NO=NN(L)                                                          BAL02260
      IF(NO.EQ.0)  GO TO 1000                                           BAL02270
      DO 999 N=1,NO                                                     BAL02280
      LL=6                                                              BAL02290
      DO 530 I=1,6                                                      BAL02300
530   FL(I)=WT(L,I,N)                                                   BAL02310
      DO 540 I=1,6                                                      BAL02315
      DO 540 J=I,6                                                      BAL02320
      LL=LL+1                                                           BAL02330
  540 PL(LL)=FL(I)*FL(J)                                                BAL02340
      DO 560 I=1,6                                                      BAL02350
      C2P(I)=0.                                                         BAL02355
  560 C1L(I)=0.                                                         BAL02360
      DO 600 J=1,6                                                      BAL02370
      DO 600 I=1,6                                                      BAL02380
  600 C1L(J)=BC(J,I)*FL(I)+C1L(J)                                       BAL02390
      DO 700 J=1,6                                                      BAL02400
      DO 700 I=7,27                                                     BAL02410
700   C2P(J)=C2P(J)+BC(J,I)*PL(I)                                       BAL02420
      DO 800 J=1,6                                                      BAL02430
      IF(PRMSEN(J).EQ.0.0)  GO TO 800                                   BAL02440
      THCK(J)=(C1L(J)+C2P(J))/PRMSEN(J)                                 BAL02450
      DELT(J)=RD(L,J,N)-THCK(J)                                         BAL02460
  800 CONTINUE                                                          BAL02470
      WRITE(6,9006) L,N,(WT(L,M,N),M=1,6),(DELT(J),J=1,6)               BAL02480
  999 CONTINUE                                                          BAL02490
 1000 CONTINUE                                                          BAL02500
       END                                                              BAL02510
      SUBROUTINE FIT1(NPT,XX,Y,AN)                                      BAL02520
      DIMENSION XX(10),Y(10),AN(2),N(5),X(9,10),A(26,3)                 BAL02530
      EXTERNAL VF1                                                      BAL02540
      N(1)=2                                                            BAL02550
      N(2)=NPT                                                          BAL02560
      N(3)=2                                                            BAL02570
      N(4)=2                                                            BAL02580
```

APPENDIX B (CONT.)

```
      N(5)=0
      DO 10 I=1,2
      A(I,3)=I
   10 A(I,1)=0.

      DO 20 L=1,NPT
      X(1,L)=XX(L)
      X(2,L)=Y(L)
   20 X(3,L)=1.

      DELT=1.0E-6
      CALL LSQSLB(N,X,A,VF1,DELT)
      DO 30 I=1,2
      AN(I)=A(I,1)
   30 CONTINUE
      RETURN
       END
      SUBROUTINE VF1(N,X,DF,A)
      DIMENSION DF(1),X(1),A(1),N(1)

*  EQUATION    Y=A(1)X+A(2)X**2

      DF(1)=X(1)
      DF(2)=X(1)**2
      X(9)=A(1)*DF(1)+A(2)*DF(2)
      RETURN
       END
      SUBROUTINE FIT2(NPT,XX,Y,AN)
      DIMENSION XX(10),Y(10),AN(1),N(5),X(9,10),A(26,3)
      EXTERNAL VF2
      N(1)=1
      N(2)=NPT
      N(3)=2
      N(4)=2
      N(5)=0
      A(1,3)=1.
   10 A(1,1)=0.0
      DO 20 L=1,NPT
      X(1,L)=XX(L)
      X(2,L)=Y(L)
   20 X(3,L)=1.
      DELT=1.0E-6
      CALL LSQSLB(N,X,A,VF2,DELT)
      AN(1)=A(1,1)
      RETURN
       END
      SUBROUTINE VF2(N,X,DF,A)
      DIMENSION DF(1),X(1),A(1),N(1)

*  EQUATION    Y=AX

      DF(1)=X(1)
      X(9)=A(1)*DF(1)
      RETURN
       END
```

APPENDIX C

DATA-REDUCTION SUBROUTINES

DATA REDUCTION SUBROUTINES
APPENDIX C

```
      SUBROUTINE BALPR(JCMPTS,BM,C1I,C1IC2)                             FORCE
C
C     PURPOSE
C        CALCULATE THE INVERSE MATRIX OF THE NORMALIZED 1ST
C        ORDER INTERACTIONS AND THE PRODUCT OF THIS INVERSE
C        MATRIX WITH THE NORMALIZED 2ND ORDER INTERACTION MATRIX
C
C     DESCRIPTION
C        JCMPTS INPUT NUMBER OF BALANCE COMPONENTS
C        BM     INPUT NORMALIZED BALANCE MATRIX
C        C1I    OUTPUT INVERSE MATRIX OF 1ST ORDER INTERACTIONS
C        C1IC2  OUTPUT PRODUCT OF C1I AND 2ND ORDER INTERACTIONS
C
C
      DIMENSION C2(126),B(6,1),C1(6,6),BM(6,27),C1I(36),C1IC2(162)      FORCE
      M=JCMPTS                                                          FORCE
      NC=M*(M+1)/2                                                      FORCE
      DO 30  J=1,M                                                      FORCE
      B(J,1)=1.0                                                        FORCE
      DO 30  K=1,M                                                      FORCE
   30 C1(J,K)=BM(J,K)                                                   FORCE
      CALL MATRIX(C1,M,6,B,0,DETERM,ID)                                 FORCE
      L=0                                                               FORCE
      DO 40 K=1,M                                                       FORCE
      DO 40 J=1,M                                                       FORCE
      L=L+1                                                             FORCE
   40 C1I(L)=C1(J,K)                                                    FORCE
      L=0                                                               FORCE
      K1=M+1                                                            FORCE
      K2=M+NC                                                           FORCE
      DO 50  K=K1,K2                                                    FORCE
      DO 50 J=1,M                                                       FORCE
      L=L+1                                                             FORCE
   50 C2(L)=BM(J,K)                                                     FORCE
***
      CALL GMPXFA(C1I,C2,C1IC2,M,M,NC)
      RETURN                                                            FORCE
       END                                                              FORCE
```

```
      SUBROUTINE ITERA(TH,LOAD,PRMSEN,C1I,C1IC2,JCMPTS,ITMAX,ACCUR,IER)   FORCE
C
C     PURPOSE
C        REDUCES THE DATA TO LOADS BY USING THE BALANCE MATRIX AND
C        ITERATING
C
C     DESCRIPTION OF PARAMETERS
C        TH     CHANNEL READINGS IN COUNTS
C        LOAD   LOAD COMPONENTS CORRECTED FOR INTERACTIONS
C               (FORCE AND MOMENT UNITS)
C        C1I    INVERSE OF NORMALIZED 1ST ORDER INTERACTIONS
C        C1IC2  PRODUCT OF C1I AND NORMALIZED 2ND ORDER INTERACTIONS
C        JCMPTS NUMBER OF BALANCE COMPONENTS
C        ITMAX  MAXIMUM NUMBER OF ITERATIONS
C        ACCUR  SMALLEST LOAD UNIT THAT CAN BE RESOLVED
C        IER    ERROR INDICATOR FOR CONVERGENCE
C               IER = 0  LOADS CONVERGED
C               IER = 1  LOADS DID NOT CONVERGE BEFORE ITMAX
C               WAS REACHED
```

```
      REAL LU(6),PRMSEN(6),TH(6),EPSO(6),P(36),C1I(36),C1IC2(162),LOAD(6 FORCE
     1),DELTA(6),ACCUR(6),EPSI(6)                                       FORCE
      M=JCMPTS                                                          FORCE
      N=M*(M+1)/2                                                       FORCE
      DO 20 I=1,JCMPTS                                                  FORCE
   20 LU(I)=PRMSEN(I)*TH(I)                                             FORCE
****
      CALL GMPXFA(C1I,LU,LOAD,M,M,1)
      L=0                                                               FORCE
      DO 60  I=1,M                                                      FORCE
      EPSO(I)=0.0                                                       FORCE
      DO 60  J=I,M                                                      FORCE
      L=L+1                                                             FORCE
      IF(I.NE.J) GO TO 59
      IF(LOAD(I)) 58,59,59
   58 P(L)=-LOAD(I)*LOAD(I)
      GO TO 60
   59 P(L)=LOAD(I)*LOAD(J)
   60 CONTINUE
      IT=0                                                              FORCE
      DO 300 NI=1,ITMAX                                                 FORCE
      IT=IT+1                                                           FORCE
   80 CALL GMPXFA(C1IC2,P,EPSI,M,N,1,     )
   90 CALL GMSBF(EPSI,EPSO,DELTA,M)                                     FORCE
  100 CALL GMSBF(LOAD,DELTA,LOAD,M)                                     FORCE
      L=0                                                               FORCE
      DO 105  I=1,M                                                     FORCE
      DO 105  J=I,M                                                     FORCE
      L=L+1                                                             FORCE
  105 P(L)=LOAD(I)*LOAD(J)                                              FORCE
  110 CALL TESTF(DELTA,ACCUR,M,ICNVG)                                   FORCE
  120 IF(ICNVG) 400,130,400                                             FORCE
C     ICNVG=0  HAS NOT CONVERGED                                        FORCE
C     ICNVG=1  ALL DELTAS ARE LESS THAN ACCUR                           FORCE
  130 CALL GMEQF(EPSI,EPSO,M)                                           FORCE
      IF(IT.EQ.ITMAX) IER=1                                             FORCE
  300 CONTINUE                                                          FORCE
  400 RETURN                                                            FORCE
       END                                                              FORCE



      SUBROUTINE GMPXFA(A,B,R,M,N,L)
C
C     PURPOSE
C        MULTIPLY TWO GENERAL MATRICES TO FORM A
C        RESULTANT GENERAL MATRIX
C
C     DESCRIPTION OF PARAMETERS
C        A   INPUT FIRST MATRIX NAME
C        B   INPUT SECOND MATRIX NAME
C        R   OUTPUT MATRIX NAME
C        M   INPUT NUMBER OF ROWS IN MATRIX A OR R
C        N   INPUT NUMBER OF COLUMNS IN A AND ROWS IN B
C        L   INPUT NUMBER OF COLUMNS IN MATRIX B OR R
C
C     METHOD
C        THE M BY N MATRIX A IS POSTMULTIPLIED BY THE N BY L MATRIX
C        B AND THE RESULT IS STORED IN THE M BY L MATRIX R.
C
```

```
C       REFERENCE
C          NASA TN D-6860
C
        DIMENSION A(1),B(1),R(1)
        IR=0
        IK=-N
        DO 30  K=1,L
        IK=IK+N
        DO 20  J=1,M
        IR=IR+1
        JI=J-M
        IB=IK
        R(IR)=0.
        DO 10  I=1,N
        JI=JI+M
        IB=IB+1
   11   R(IR)=R(IR)+A(JI)*B(IB)
   10   CONTINUE
   20   CONTINUE
   30   CONTINUE
        RETURN
         END
```

```
        SUBROUTINE GMSBF(A,B,R,MN)
C
C       PURPOSE
C          SUBTRACT ONE GENERAL MATRIX FROM ANOTHER TO FORM
C          A RESULTANT GENERAL MATRIX
C
C       DESCRIPTION OF PARAMETERS
C          A  INPUT NAME OF FIRST MATRIX
C          B  INPUT NAME OF SECOND MATRIX
C          R  OUTPUT MATRIX NAME
C          MN INPUT NUMBER OF ELEMENTS IN MATRIX A, B, OR R
C
C       METHOD
C          EACH ELEMENT OF MATRIX B IS SUBTRACTED FROM THE
C          CORRESPONDING ELEMENT OF MATRIX A AND THE RESULT IS
C          PLACED IN THE CORRESPONDING ELEMENT OF MATRIX R
C
C       REFERENCE
C          NASA TN D-6860
C
        DIMENSION A(1),B(1),R(1)
        DO 10  IJ=1,MN
        R(IJ)=A(IJ)-B(IJ)
   10   CONTINUE
        RETURN
         END
```

```
        SUBROUTINE TESTF(A,H,MN,LF)
C
C       PURPOSE
C          TEST THE ABSOLUTE VALUE OF EACH ELEMENT OF MATRIX A TO
C          DETERMINE IF IT IS LESS THAN OR EQUAL TO THE
```

```
C         CORRESFONDING ELEMENT OF MATRIX B
C
C      DESCRIPTICN OF FARAMETERS
C         A  INPLT FIRST MATRIX NAME
C         B  INPLT SECCND MATRIX NAME
C         MN INPUT NUMBER OF ELEMENTS IN MATRIX A CR B
C         LE OUTPUT COMPARISON OF MATRICES A AND B
C
C      METHOD
C         IF ABSCLUTE VALUE OF A(IJ) IS LESS THAN CR EQUAL TO
C         B(IJ) FOR ALL IJ = 1, 2,....., MN THEN LE = 1
C         OTHERWISE, LE = 0.
C
C      REFERENCE
C         NASA TN D-6860
C
       DIMENSION A(1),B(1)
       LE=0
       DO 10  IJ=1,MN
       IF(ABS(A(IJ))-B(IJ)) 10,10,20
    10 CONTINUE
       LE=1
    20 RETURN
        END




       SUBROUTINE GMEQF(A,R,MN)
C
C      PURPOSE
C         EQUATE ONE GENERAL MATRIX TO ANOTHER GENERAL MATRIX
C
C      DESCRIPTICN OF FARAMETERS
C         A  INPLT MATRIX NAME
C         R  OUTPUT MATRIX NAME
C         MN INPLT NUMBER OF ELEMENTS IN MATRIX A CR R
C
C      METHOD
C         EACH ELEMENT OF MATRIX R IS SET EQUAL TO THE
C         CORRESPONDING ELEMENT OF MATRIX A
C
C      REFERENCE
C         NASA TN D-6860
C
       DIMENSION A(1),R(1)
       DO 10  IJ=1,MN
       R(IJ)=A(IJ)
    10 CONTINUE
       RETURN
        END
```